**Report on SageMath Tool and its Functionalities**

**Introduction**

SageMath (or simply Sage) is an open-source mathematics software system that integrates a wide variety of mathematical tools and libraries into one cohesive platform. It is designed to provide an alternative to proprietary software such as Mathematica, Maple, and MATLAB. SageMath encompasses a wide range of mathematical functions, including algebra, calculus, number theory, cryptography, and much more. The aim of SageMath is to offer an accessible, open-source alternative for both students and professionals in mathematics and related fields.

**Key Features and Functionalities of SageMath**

1. **Algebra**
   * **Polynomial Operations**: SageMath allows users to perform operations on polynomials, such as factorization, finding roots, and polynomial division. It supports univariate and multivariate polynomials.
   * **Commutative and Non-Commutative Algebra**: SageMath provides functionalities for both commutative and non-commutative algebraic structures such as groups, rings, and modules.
   * **Simplification**: Users can simplify algebraic expressions, including rational expressions and algebraic fractions.
2. **Calculus**
   * **Differentiation and Integration**: SageMath supports symbolic differentiation and integration, both definite and indefinite, for a wide variety of mathematical expressions.
   * **Limit and Series Expansion**: SageMath is capable of computing limits of functions, including asymptotic behavior. It can also handle series expansions (e.g., Taylor series) for complex functions.
   * **Solving Differential Equations**: SageMath provides tools to solve ordinary differential equations (ODEs) symbolically or numerically.
3. **Linear Algebra**
   * **Matrix Operations**: SageMath provides a wide range of matrix operations, such as matrix multiplication, determinants, eigenvalues, and eigenvectors.
   * **Solving Systems of Linear Equations**: It supports solving systems of linear equations with methods like Gaussian elimination and matrix inversion.
   * **Vector Spaces**: SageMath can work with vector spaces, subspaces, and linear transformations.
4. **Number Theory**
   * **Prime Factorization**: SageMath includes efficient algorithms for factoring large numbers and checking for primality.
   * **Modular Arithmetic**: It offers tools to work with modular arithmetic, including modular inverses, congruences, and Chinese Remainder Theorem.
   * **Cryptography**: SageMath has built-in functionality for implementing cryptographic algorithms such as RSA, elliptic curve cryptography, and digital signatures.
5. **Graph Theory**
   * **Graph Construction**: Users can construct and manipulate both directed and undirected graphs, including weighted and unweighted graphs.
   * **Graph Algorithms**: SageMath includes algorithms for finding shortest paths, computing graph connectivity, and generating spanning trees, among others.
   * **Graph Drawing**: SageMath provides basic graph visualization features, enabling users to visually explore graph structures.
6. **Statistics**
   * **Descriptive Statistics**: SageMath can compute measures like mean, median, mode, variance, and standard deviation for data sets.
   * **Probability Distributions**: SageMath supports a variety of continuous and discrete probability distributions (e.g., binomial, normal, Poisson).
   * **Hypothesis Testing**: The tool includes statistical tests like t-tests, chi-square tests, and linear regression analysis.
7. **Plotting and Visualization**
   * **2D and 3D Plotting**: SageMath can generate 2D and 3D plots for functions, data, and geometric objects. Users can create graphs, contour plots, surface plots, and parametric plots.
   * **Interactive Plotting**: SageMath supports interactive plotting, which allows users to manipulate the plots dynamically.
   * **Customization**: SageMath includes various options for customizing plots, including colors, labels, axis properties, and more.
8. **Numerical Computation**
   * **Floating Point Arithmetic**: SageMath supports numerical solutions to equations and systems that cannot be solved symbolically. It uses various numerical methods to find approximate solutions.
   * **Optimization**: SageMath includes tools for solving optimization problems, such as linear programming and nonlinear optimization.
   * **Numerical Integration**: Users can compute numerical integrals using methods such as Simpson's rule, trapezoidal rule, and others.
9. **Symbolic Computation**
   * **Symbolic Expressions**: SageMath allows manipulation of symbolic expressions in terms of variables, constants, and functions. This makes it useful for solving equations and simplifications.
   * **SymPy Integration**: SageMath integrates SymPy, a symbolic mathematics library, allowing users to perform symbolic algebra, calculus, and other operations.
   * **Algebraic Geometry**: It also has tools for symbolic computations in algebraic geometry, such as ideal theory and Gröbner bases.
10. **Development and Extensibility**
    * **Python Integration**: SageMath is built on top of Python, which means it can be extended using Python's capabilities. Users can write custom functions, loops, and other programming constructs.
    * **Interface with Other Software**: SageMath is designed to integrate with other open-source and proprietary software systems like Maxima, GAP, R, and MATLAB. This helps users take advantage of specialized tools while working in a unified environment.
    * **Jupyter Notebooks**: SageMath supports integration with Jupyter notebooks, allowing users to create dynamic, interactive documents that combine code, visualizations, and explanations.
11. **Numerical Solvers**
    * **Roots of Equations**: SageMath provides various methods for finding roots of equations, both in the real and complex domains.
    * **Nonlinear Equations**: Tools for solving nonlinear equations numerically are available, using methods like Newton’s method or bisection.
    * **Eigenvalue Problems**: The software includes efficient algorithms for solving eigenvalue problems, both for real and complex matrices.

**Use Cases**

1. **Academic and Educational Use**: SageMath is widely used in academia for teaching and learning purposes. Its easy access to a broad range of mathematical topics makes it suitable for students in various fields, from basic algebra to advanced number theory and cryptography.
2. **Research and Industry Applications**: Researchers in fields like physics, engineering, economics, and cryptography use SageMath for modeling, simulations, and problem-solving. Its integration with other scientific tools adds flexibility and power for specialized applications.
3. **Open Source Community**: As an open-source tool, SageMath has a vibrant community that contributes to its development. Researchers, developers, and hobbyists work together to improve its functionality and expand its feature set.